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Book Review
Lozi Mappings: Theory and Applications
by Zeroualia Elhadj, CRC Press, 2013

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Abstract

When, more than two years ago, Prof. Zeroualia Elhadj informed me of his willing to write a book on what is known as "Lozi map" since the Misiurewicz's communication in the congress organized by the New York Academy of Science, 17-21 December 1979, I warned the task was not straightforward because hundreds of articles were published on this topic in thirty years. These papers were scattered in various fields of research, not only in mathematics (dynamical systems), but in physics, computer science, electronics, chemistry, control science and engineering, etc.

Keywords: Lozi Mappings, mathematical theory, real world-applications

Nevertheless, he eventually collected and scrutinized more than one thousand papers before completing this outstanding book "Lozi Mappings: Theory and Applications". The outcome of his enquiry is tremendous. Every aspect of the mathematical properties of this map of the plane (and its generalizations) is analyzed. The results are classified and systematized. Moreover, in order to make easy the comprehension for a fresh reader, the book begins with a comprehensive review of hyperbolicity, ergodicity and chaos. Once the background is clearly posed the reality of chaos in the Hénon mappings is examined, after that the survey on Lozi mappings begins.

Responding to the kind invitation of Prof. Zeroualia Elhadj to write an introduction, I take the opportunity to introduce some personal views not only on the matter of chaotic

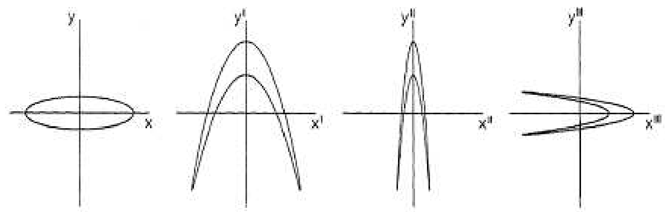

Figure 1: The initial area a maped by T' into b, then by T'' into c, and finally by T''' into d.

systems, but also on the current evolution of mathematics and some aspects of the live of one researcher in mathematics.

In the human life it is not so easy to recall a particular day. Thirty five years after the pinpoint moment I had the idea to substitute the quadratic term in the Hénon map by an absolute value I can remember the exact date because it took place during the talk of the presentation of the thesis of A. Intissar on June 15th 1977 around 11 a.m (I checked recently the date). In these days the department of mathematics of the university of Nice (later called university of Nice-Sophia Antipolis) was a small community and every one attended the presentation of each Ph.D. thesis. Hence I was not very concerned by the talk and contrarily I was thinking thoroughly to the strange structure of the Hénon map that my colleague Gérard Iooss told me about, few days before, during the "International Conference on Mathematical Problems in Theoretical Physics" that took place at the university of Roma, Italy (June 6-15), we partially attended together. The opening talk of this conference given by David Ruelle (Dynamical Systems and Turbulent Behavior) emphasized the importance of such a simple discrete model in the study of turbulence (this is not recognized today). At that time I occupied the position of "Attaché de Recherches" at C.N.R.S. (Centre National de la Recherche Scientifique) after my Ph. D. thesis on numerical analysis of bifurcation problems (the first thesis on bifurcation theory in France, presented on April 25th, 1975). I was mainly interested in discretization problems and finite element methods, in which nonlinear function are approximated by piecewise linear ones. I tried to apply my background to the quadratic map introduced by Michel Hénon few months ago, in order to obtain a better amenable map for analytical treatment. In Figure 1 of his publication (reproduced here) there is a clear explanation of the folding and stretching process which led him to the formula of the map.

The area b on the Figure is bounded by two parabolas generated by the formula: $T' : x' = x, y' = y + 1 - ax^2$ applied to the initial area a. Drawing on a paper sheet the shape of this area, I embedded it in another area bounded by four line segments which eventually reminded me the graph of the absolute value function. I substituted then $L' : x' = x, y' = y + 1 - a\,|x|$ to T'. Soon after the end of the presentation I went to my office situated on the upper floor of the seminar room to test the idea on the Hewlett-Packard 9820 calculator linked to the HP 9862 plotter I used to promote computer science for teachers in the classroom at the Institute of Research in Educational Mathematics (I.R.E.M). Even if the parameter value giving the "classical" Hénon strange attractor (i.e., $a = 1.4, b = 0.3$) provides also a strange attractor for this new mappings, after few tests I shifted it to $a = 1.7, b = 0.5$ in order to obtain a more striking picture of the strange attractor studied in this book. Back to the lunch which celebrated the completion of the thesis I showed the figure to Gérard Iooss and also to Alain Chenciner who encouraged me latter to publish the formula, (the genuine article comes from the presentation I gave during a conference on dynamical systems in July 1977 in Nice). In the following days I was convinced that few weeks would be enough to explain the structure of such an attractor basically composed of line segments. But the task proved more difficult than expected (mainly because contrary to as Michal Misiurewicz did, I did not limited the extend of the parameter value for the study). In the next years I attended two meetings on iteration theory: the first one on May 21-23, 1979 at La Garde Freinet (a small town in the south of France) where Michel Hénon was also present and where Michal Misiurewicz, after some questions at the end of my talk (the purpose of which was the computation of homoclinic points of the map), came to the blackboard to give to the assistance some clues of the forthcoming result of the New York meeting. The second meeting is a summer school in physics on July 1979 in Cargèse (Corsica) in the proceedings of which I eventually published the article entitled: "Strange attractors: a class of mappings of $\mathbb{R}^2$ which leaves some Cantor Set invariant". In this paper I used the genuine non differentiable map in order to prove the existence of one homoclinic point for a smooth version of the Lozi map and then applying a theorem of Stephen Smale I proved the existence of an invariant Cantor set. After that, took place the congress organized by the New-York Academy of Science where I am proud I shook the hand of Edward Lorenz, the father of strange attractors and I listen with a mix of anxiety and curiosity the first proof of existence of a strange attractor for an analytically given map of the plane. After the Misiurewicz's work, hundreds of papers were published on countless aspects of this strange attractor as it is cleverly showed in this book.

Now, if we go back in thought in the late 70's, in some aspects, life was very different than nowadays. There was no personal computer (M. Hénon used one of the only two computers of the university of Nice, a IBM 7040, in order to plot the figure of his original paper), no Internet, no wireless phone. Also, communications between researchers were done through slow post office mail, travels by air were very expensive, limiting personal contact between researchers in the west countries. Moreover, the Berlin's wall was still standing. James Yorke which coined the term of chaos in his famous paper with his student Tien-Yien Li "Period three implies Chaos" in 1975 was unaware of Alexander Sharkovkii's theorem published in Russian in 1964 displaying more penetrating results on periodic orbits (however essential notion as sensitive dependence on initial conditions is only introduced in the paper of Li and Yorke). The technical progress in thirty years is dramatic in every aspect of all the days life. In contrast, mathematics is progressing very slowly. Near my entire professional life of mathematician has been needed to see published results I expected proved in few months. News results as for example: "topological entropy for the Lozi maps can jump from zero to a value above 0.1203 as one crosses a particular parameter and hence it is not upper semi-continuous in general" (I. B. Yildiz), or: "certain Lozi-like maps have the orbit-shifted shadowing property" (A. Sakurai) are continuously published every year.

A tentative title first chosen to the book by Prof. Elhadj Zeraoulia: "the power of chaos" reflects a part of what one can observe in numerous publications of last years. If on one hand new theoretical important results are still regularly found as said above, on the other hand applications of Lozi map are soaring, in engineering, computers, communications, control, medicine and biology and especially in the domain of evolutionary algorithms. In the last chapter of the book, due to the limitation of the number of pages, only some real-world applications are given. However, they allow the reader to get an idea of the power that holds the mastery of chaos generated by the Lozi mappings.

In the scope of evolutionary algorithms the use of chaotic sequences instead of random ones has been introduced ten years ago by Caponetto, *et al.* Several traditional chaotic maps in 1 or 2-dimensions are used. The difference among them is based on the two main differences: the shape of the invariant measure and the robustness with respect to numerical computation. Lozi map is recognized to show better performance due to his shadowing property. These algorithms are particularly efficient in global optimization problems. Since 1975, when I was a young assistant professor, I had a disappointed passion for years in the search of classical algorithm for solving such problems. What is funny nowadays is to notice how a map I introduced for an entire different aim is routinely used now to solve this problem.

Finally thanks to the work of Prof. Elhadj Zeroualia, I can go back and look at how a very small idea has blossomed into an area that still fascinates me: mathematics.