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Evidence of Noisy Chaotic Dynamics in the Returns of Four Dow Jones Stock Indices

John Francis T. Diaz
Chung Yuan Christian University, College of Business, Chung-li City, Taiwan
e-mail: di.jiang@cycu.edu.tw

Abstract

This research finds evidence of noisy chaotic properties in the returns of four Dow Jones indices, based on three tests of non-linearity and chaos. The study uses an average of 24,815 data points to correctly simulate chaos in financial time-series. The data consists of the Dow Jones Industrial Average (29,229 observations); Dow Jones Transportation Average (29,121 observations); Dow Jones Utility Average (21,150 observations) and the Dow Jones Composite Average (19,906 observations). The a) Brock, Dechert, and Scheinkman (BDS) test indicates that most of the Dow Jones indices are not *iid* series, except for the filtered residuals from the GARCH of the Dow Jones Utility Average. The b) rescaled range analysis shows that after scrambling the data, all Hurst exponents are above 0.5, and a trend-reinforcing property, which helps in the conclusion of having a chaotic process. Lastly, the c) correlation dimension analysis complements the initial findings and concludes the presence of a high dimensional noisy chaotic structure in the four Dow Jones indices.

Keywords: Stock returns and volatility, Dow Jones indices, Statistical Physics, Noisy chaotic process

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1 Introduction

The straightforward solutions given by linear models are becoming inadequate with the growing complexities of financial time-series. Investors' treatment of risk and expected returns, strategic interactions between financial market participants and the way by which information is integrated into security prices, all behave in a nonlinear manner [1]. The authors conclude that the natural tendency in modeling financial time-series is to consider nonlinear dynamics. The importance of investigating nonlinearities in time-series data provides a hint on the hidden structure of the data, which makes it easier to distinguish between their random and chaotic properties [2]. Econometric models related to these two tendencies can be both stochastic and deterministic in nature. The study of [3] provided an initial evidence on the existence of chaos in financial markets. The author explained that a chaotic structure is a nonlinear system that appears to be random in nature making it easily misinterpreted as a random process by linear econometric method.

Chaos as an area of Econophysics has been starting to be recognized in the literature because of the availability of longer time-series data, which according to [4], enhances the accuracy of results and recommended a volume of at least 5,000 observations to sufficiently detect deterministic dynamics. The authors studied the S&P Composite Price Index using 16,127 observations and concluded that the S&P 500 showed strong evidence of chaos based on Grassberger-Procaccia's correlation dimension measurement with nonlinear noise filtering and a surrogate technique. A year ago, [5] used daily observations of 7,917 data points and observed nonlinear properties on the returns of the Swedish Stock Index. In [6], the author used a much higher frequency data of 27,523 continuous observations of the Dow Jones Industrial Average and found that long-range correlations as shown by fractal fluctuations are related to quantum-like chaos. Although the minimum number set by [4] was not strictly followed, a recent study of [7] still found conclusive evidences of nonlinearities and chaos in the returns of the main stock market indices of Czech Republic with 4,369 returns, Hungary with 4,577 observations and Poland using 4,575 data points.

Following the recommendation of having a longer time-series to identify deterministic processes, this paper was motivated to study the oldest group of stock indices still in use today, the four Dow Jones stock indices. Although the oldest index in the family is the Dow Jones Transportation Index (DJTA), which was established in 1884, the Dow Jones Industrial Average (DJIA) is the best known index, which was created in 1896. With the DJTA together with the DJIA, also experiencing record highs during the first quarter of 2013, economists contend that the US economy might be experiencing growth even in a subtle manner. This research is also interested in studying the relatively younger indices, the Dow Jones Utility Average (DJUA) and Dow Jones Composite Average (DJCA), which was created in 1929 and 1939, respectively.

This research examines evidences on the possibility of finding nonlinearities, particularly chaotic tendencies of the four Dow Jones stock indices. The persistence and nonlinear properties of the DJIA using different time-series have been established in the literature (e.g., [8-12]), but none of these have carefully studied the nonlinear tendencies of the other Dow Jones indices, specifically its chaotic properties. Another unique contribution of this paper is the sole consideration of a very long time-series without structural breaks in the data. The minimum requirement of atleast 5,000 daily data limited the paper on having structural breaks, because the interval of some crises and possible structural breaks occur in less than 5,000 daily observations [4].

Three different approaches in testing nonlinear and chaotic properties have been already established in the literature, namely, Brock, Dechert, and Scheinkman (BDS) test of [13], Rescaled Range (R/S) analysis originated by [3], and the Correlation Dimension (CD) analysis of [14]. This study contributes to the literature of applying Statistical Physics methodologies to Economics or what is now known as the field of Econophysics, and aims to 1) provide additional evidence on the nonlinearities in financial time-series through examining chaotic properties of the DJIA, DJTA, DJUA and DJCA returns. This research also wants to 2) challenge the validity of the efficient market hypothesis (EMH) of [15], which has been the explanation on the stochastic attributes of financial time-series. To the best of the author's knowledge, no research yet has been done to determine chaos in the returns of these four Dow Jones indices. The strength of studying a relatively longer time-series can benefit the investing community in understanding the Dow Jones indices market behavior and provides a considerable amount of knowledge for both academicians and researchers in providing potential avenues for future research.

The paper is structured as follows. Section 2 presents the data and methodology of the three tests of Chaos, namely, BDS test, R/S analysis and the CD analysis; Section 3 interprets empirical findings; and Section 4 provides the conclusion.

2 Data and Methodology

This research analyzes daily closing prices of the four Dow Jones indices from the Federal Reserve Bank of St. Louis database until February 26, 2013. Although the oldest stock index, the DJTA was created in 1884, the oldest available data provided started from May 27, 1896 and has a total of 29,229 observations for DJIA; DJTA began from October 27, 1896 and has 29,121 data points; DJUA started from January 3, 1929 and has 21,150 observations; and DJCA began from January 3, 1934 with 19,906 data points. The series of returns were computed as, $y_t = 100\,(\log p_t - \log p_{t-1})$ where p_t represents the Dow Jones index price at time t.

2.1 Chaos methodologies

According to Peters (1994), the existence of a fractal dimension and sensitivity dependence on initial conditions are the two necessary conditions for a process to become chaotic. The research utilizes three different approaches in testing the chaotic dynamics of the four Dow Jones indices. The detailed three methodologies presented in this research are as follows:

2.1.1 Brock, Dechert, and Scheinkman test

The BDS test [13] is a way to detect dependency in financial time series. The test gives delineation between a random series from deterministic chaos or from nonlinear stochastic series. However, the BDS test has a low power against the autoregressive (AR) andautoregressive conditional heteroscedasticity (ARCH) models [3]. To compensate this shortcoming, this study pre-filters the financial time series with linear filter like the autoregressive moving average (ARMA) and a nonlinear filter like generalized autoregressive conditional heteroscedasticity (GARCH) before proceeding with the BDS test. These processes eliminate linearity from the returns, and any dependence found in the residuals.

The sample correlation integral is the basis in calculating the BDS test statistic, which can be defined as:

$$C_N\left(l,T\right) = \frac{2}{T_N\left(T_N-1\right)} \sum_{t<s} I_l\left(x_t^N, x_s^N\right), \tag{1}$$

where $T_N = T - N + 1$.
The correlation integral of the time-series is dependent on a sequence $x_t := 1, ...T$ of observations which are independent and identically distributed *(iid)*, and N-dimensional vectors $\left[x_t^N = (x_t, x_{x+1}\right]$ called the "N-histories".

The null hypothesis $\{x_t\}$ under the test is that the increments of the data time-series is *iid* with a non-degenerative density F, $C_N(l,T) \rightarrow C_1(l)^N$ with probability of one, as $T \rightarrow \infty$, for any fixed N and l; and that $\sqrt{T}\left[C_N(l,T) - C_1(l,T)^N\right]$ has a normal distribution with zero mean and variance [13].

$$\sigma_N^2\left(l\right) = 4\left[K^N + 2\sum_{j=1}^{N-1} K^{N-1}C^{2j} + (N-1)^2C^{2N} - N^2KC^{2N-2}\right], \tag{2}$$

where $C = C(l) = \int[F(z{+}1){-}F(z{-}1)]dF(z), K = K(l) = \int\int[F(z+1) - F(z-1]^2dF(z)$.

The term $C_1(l,T)$ is a consistent estimate of $C(l)$, and

$$K\left(l,T\right) = \frac{6}{T_N\left(T_{N-1}\right)(T_{N-2})} \sum_{t<s<r} I_l(x_t,x_s)I_l(x_s,x_r). \tag{3}$$

Furthermore, $\sigma_N(l)$ can be also estimated by $\sigma_N(l,T)$, which $C_1(l,T)$ and $K_1(l,T)$ can be put in place of $C(l)$ and $K(l)$ in the equation, because Eq.(3) is also a consistent estimate of $K(l)$. Following a normal distribution, the BDS test structure can be completed as follows:

$$w_N\left(l,T\right) = \sqrt{T}\left[C_N(l,T) - C_1(l,T)^N\right]/\sigma_N(l,T), \tag{4}$$

where $\sigma_N(l,T)$ denotes the standard deviation of the correlation integrals.

2.1.2 Rescaled Range analysis: Hurst exponent

The R/S statistic or the so-called rescaled range defines the R/S analysis, which was developed by [16]. The initial rescaled range procedure was improved by [17] and [18], which has the limitation of determining range dependencies without discriminating between short and long dependencies in the series of data [19]. The improvements made the modified R/S analysis to remove short-term dependencies and also able to identify long term dependencies. This research initially transformed the financial time-series into logarithmic returns and is given by:

$$S_1 = ln(P_t/P_{t-1}), \tag{5}$$

where S_t = logarithmic returns at time t, and P_t = Dow Jones price index at time t. The S_t series undergoes a process called pre-whitening to minimize the effect of linear dependency and non-stationarity which is calculated as follows:

$$S_t = \alpha + \beta S_{t-1} + \varepsilon_t, \tag{6}$$

where S_{t-1} denotes the logarithmic return at time period $t-1$ and ε and β exhibit the parameters to be estimated and ε_t represents the residual.

The data is separated into A adjacent sub-periods of length n, such that$A \times n = N$, where N represents the extent of the series N_t , which is similar to the processes of [20] and [21]. Each sub-period is defined as I_a, $a = 1, 2, 3, , A$. The time series in I_a is marked $N_{k,a}$, $k = 1, 2, 3, , n$ and the average value e_a for each I_a of length n is

$$e_a = \left(\frac{1}{n}\right) \times \sum_{k=1}^{n} N_{k,a}. \tag{7}$$

$$R_{Ia} = \max(X_{k,a}) - \min(X_{k,a}), where 1 \leq k \leq n, 1 \leq a \leq A, \tag{8}$$

The range R_{Ia} denotes the difference between the maximum and minimum value $X_{k,a}$ within each sub-period I_a, and expressed as:
$X_{k,a} = \sum_{i=1}^{k}(N_{i,a} - \varepsilon_a)$, $k = 1, 2, 3, ..., n$ exhibits the elements for each sub-period of departures from the mean value. The R/S analysis requires that R_{Ia} should be normalized by dividing the sample by the standard deviation S_{Ia} corresponding to it and is computed below:

$$S_{Ia} = \left[\frac{1}{n} \times \sum_{k=1}^{n}(N_{k,a} - \varepsilon_a)^2\right]^{0.50} \tag{9}$$

The average R/S value for the length n is computed as:

$$\left(\frac{R}{S}\right)_n = \left(\frac{1}{A}\right) \times \sum_{a=1}^{A}(R_{Ia}/S_{Ia}). \tag{10}$$

The final procedure in the analysis is the application of an ordinary least squares (OLS) regression with $log(n)$ as the independent variable and $log(R/S)$ as the dependent variable. The H exponent can have the following values: $H = 0.5$, which denotes the Dow Jones series is a random walk; $0 \leq H < 0.5$, which represents an anti-persistent series, or a medium memory; and $0.5 < H < 1$, which exhibits a persistent series, or a series with long-memory. The R/S analysis computation can be derived from the expected values of the R/S statistics:

$$E\,(R/S) = \left[\left(\frac{n-0.5}{n}\right) \times \left(\frac{n \times \pi}{2}\right)\right]^{-0.50} \times \sum_{r=1}^{n-1} \sqrt{\frac{(n-r)}{r}}. \tag{11}$$

The Hurst exponent,H is computed from the slope of the regression of $E(log(R/S)_n$ on $log(n)$. The variance of the Hurst exponent can be shown as:

$$Var(H)n = \frac{1}{T} \tag{12}$$

where T denotes the total number of observations in the series.

2.1.3 Correlation Dimension Analysis

The CD method as introduced by [14] differentiates deterministic and stochastic time series. The methodology examines the amount of complexity of a time-series data, which helps in determining possible signs of chaos in the four Dow Jones indices. Based on the recommendation of [14] and [3], the CD analysis requires the initial filtering of the observations through the ARMA and GARCH processes to eliminate possible problems of autocorrelation and conditional heteroscedasticity, respectively. The filtering process is followed by creating n-histories of the filtered data, the process is shown below:

$$1 - history : x_t^1 = x_t, \tag{13}$$

$$2 - history : x_t^2 = (x_{t-1}, x_t), \tag{14}$$

$$n - history : x_t^n = (x_{t-n+1,\ldots,}, x_t). \tag{15}$$

where n-history represents a particular point in the n-dimensional space.

This is followed by the calculation of correlation integral to define the correlation dimension which can be shown as:

$$C_n(\varepsilon) = lim_{T\to\infty} \sharp \left\{(t,s), 0 < t, s, < T : \|x_t^n - x_s^n\| < \varepsilon\right\}/T^2, \tag{16}$$

where $\sharp$ correspond to the number of points in the set, and $\|\ \|$ represents the sup- or max- norm making the correlation integral $C_n(\varepsilon)$ the fraction of pairs (x_s^n, x_t^n), which are close to each other, based on the limit:

$$max_{i=0,\ldots,n-1} \left\{|x_{s-i} - x_{t-i}|\right\} < \varepsilon. \tag{17}$$

The last step calculates for the slope of $logC_n(\varepsilon)$ on $log(\varepsilon)$ for small values of ε with the following equation:

$$v_n = lim_{\varepsilon\to 0} logC_n(\varepsilon)/log\varepsilon. \tag{18}$$

Deterministic chaos behavior is present if the embedding dimension increases while the value of correlation dimension (v_n) does not converge to a stable value. A stochastic chaos is determined if the correlation dimension increases without any bound.

3 Empirical Results

Table 1 shows that the four Dow Jones indices have positive returns with the DJCA posting the highest average returns of 1% for the whole data sample, followed by the DJIA

(0.9%), DJTA (0.7%) and the DJUA (0.3%) has the lowest average returns. Although the DJCA index posted the highest returns, it has the lowest volatility with 0.428 standard deviation. The DJTA which posted the highest dispersion of 0.562 is only third in the ranking of average returns. This paper concludes that the Modern Portfolio Theory [22], stating that a higher risk is compensated with higher returns, does not conform with the chosen data sets. All of the four Dow Jones indices are negatively skewed and have leptokurtic distributions. The Jarque-Bera statistic for residual normality shows that the indices returns are under a non-normal distribution assumption.

Table 2 shows the filtering done by this study by initially establishing the stationarity of the data through the Augmented Dickey-Fuller (ADF) test. The orders of the ARMA, ARMA residual and GARCH residual models use the minimum values of the Akaike Information Criterion. Most of the Dow Jones indices data period passed the serial correlation examination based on the results of the Lagrange Multiplier test. This study utilizes the ARCH-LM process to test heteroscedasticity problem and shows that we can apply GARCH filtering models for each of the designed periods, because the null hypothesis was rejected. The final test for ARCH effect showed that all data sets have already constant variance from the GARCH residual models.

Table 3 illustrates that the BDS statistics are significant for most values of ε/σ from 0.5 to 2.0, and m ranging from 2 to 6 for the filtered index returns, ARMA and GARCH residuals for all the identified data sets, except for the DJUA. The residuals of the utility index from the GARCH model failed to reject the null hypothesis of *iid* observations. The rejection of the null hypothesis can have three major possibilities of having the form of non-stationarity, linear serial dependence, or non-linear dependence, which can be either chaotic or stochastic. Earlier we have already established the stationarity of the data through the ADF test, and its nonlinearity through the ARMA and GARCH filtration process. Therefore, this study can conclude that the Dow Jones Indices are not *iid* or not pure random series, and conventional linear methodologies are not appropriate for their analysis. In earlier studies, [23] and [21] have the same findings of non-stochastic processes in the returns of S&P 500 cash index and FTSE index, respectively; and [24] and [25] regarding the chaotic properties of the Istanbul Stock Exchange Index, and the real estate investment trusts (REITs) and the Russell 2000 stock index, respectively. This study does not conform to the EMH [15], because the weak-form efficiency of the four Dow Jones indices were not validated.

The BDS test cannot conclude the *iid* properties for all the GARCH residuals of DJUA, wherein the presence of significant result cannot be discounted and may hint a possibility of having a stochastic process. Since BDS test is just the beginning in testing for chaos and cannot exactly determine chaotic properties in the Dow Jones indices, this literature conducts further tests and utilizes the R/S and correlation dimension analyses to supplement this initial examination.

Table 1: The Sample Size and Periods of the four Dow Jones indices

Dow Jones indices returns	**Start of Data**	**Obs.**	**Mean**	**Std.Dev.**	**Skew**	**Kurt.**	**J-Bera**
Dow Jones Industrial Average(DJIA)	May 27, 1896	29,229	0.009	0.503	-0.834	27.572	738745.4***
Dow Jones Transportation Average(DJTA)	October 27,1896	29,121	0.007	0.562	-0.193	16.707	228142.5***
Dow Jones Utility Average(DJUA)	January 3,1929	21,150	0.003	0.506	-0.270	27.458	527420.2***
Dow Jones Composite Average(DJCA)	January 3,1934	19,906	0.010	0.428	-0.912	32.908	744676.9***

Source:Federal Reserve Bank of St. Louis database

Note: *,** and *** are significant at 10, 5, and 1% levels, respectively; p-values are in parentheses.

Table 2: Summary Statistics of Unit Root test, and ARMA-GARCH filtering

DJ Indices	**ADF**	**ARMA**	**AIC**	**LM-test**	**ARMA Res.**	**AIC**	**LM-test**	**ARCH-LM**	**GARCH Res.**	**AIC**	**ARCH-LM**
DJIA	-82.435***	(2,2)	1.461	9.009**	(1,1)	1.460	0.062	1576.038***	(2,2)	1.013	53.640
DJTA	-81.610***	(1,2)	1.680	0.004**	(2,1)	1.679	0.267	2035.287***	(2,2)	1.229	1.781
DJUA	-61.752***	(2,2)	1.473	0.003**	(2,2)	1.468	1.008	3791.541***	(2,2)	0.627	3.714
DJCA	-80.372***	(2,2)	1.136	18.530**	(2,2)	1.133	1.705	1115.915***	(2,2)	0.788	5.367

Note: *,** and *** are significant at 10, 5, and 1% levels, respectively; p-values are in parentheses.

Table 3: BDS test for the Dow Jones Indices returns and residuals

DJIA	**DJIA returns**				**ARMA residuals**				**GARCH residuals**			
ε/σ	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
2	0.012***	0.022***	0.021***	0.015***	0.012***	0.022***	0.021***	0.014***	-0.001***	-0.001***	-0.001***	-0.001*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.006)	(0.002)	(0.005)	(0.068)
3	0.013***	0.042***	0.047***	0.037***	0.013***	0.042***	0.047***	0.037***	-0.000***	-0.002***	-0.002***	-0.001**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.007)	(0.002)	(0.002)	(0.045)
4	0.010***	0.053***	0.072***	0.061***	0.010***	0.053***	0.072***	0.061***	-0.000***	-0.002***	-0.003***	-0.002**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.008)	(0.001)	(0.001)	(0.026)
5	0.007***	0.056***	0.093***	0.085***	0.007***	0.056***	0.093***	0.085***	-0.000**	-0.001***	-0.003***	-0.002**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.032)	(0.004)	(0.004)	(0.046)
6	0.004***	0.056***	0.109***	0.108***	0.004***	0.056***	0.109***	0.108***	-0.000	-0.001**	-0.002**	-0.002
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.215)	(0.045)	(0.027)	(0.143)

Note: *,** and *** are significant at 10, 5, and 1% levels, respectively; p-values are in parentheses.

DJTA	**DJTA returns**				**ARMA residuals**				**GARCH residuals**			
ε/σ	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
2	0.013***	0.025***	0.022***	0.016***	0.013***	0.025***	0.023***	0.016***	-0.001***	-0.002***	-0.001***	-0.001***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	(0.004)
3	0.015***	0.046***	0.050***	0.039***	0.015***	0.046***	0.050***	0.039***	0.000***	-0.002***	-0.003***	-0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
4	0.011***	0.057***	0.076***	0.063***	0.011***	0.057***	0.076***	0.064***	0.000***	-0.002***	-0.004***	-0.004***
	(0.00 0)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
5	0.008***	0.061***	0.096***	0.088***	0.007***	0.061***	0.097***	0.089***	0.000***	-0.002***	-0.004***	-0.004***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
6	0.005***	0.060***	0.112***	0.111***	0.005***	0.060***	0.113***	0.112***	0.000**	-0.001***	-0.003***	-0.004***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.018)	(0.005)	(0.001)	(0.001)

Note: *,** and *** are significant at 10, 5, and 1% levels, respectively; p-values are in parentheses.

(continued)

DJUA	DJUA returns				ARMA residuals				GARCH residuals			
ε/σ	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
2	0.031***	0.043***	0.032***	0.021***	0.031***	0.043***	0.032***	0.021***	0.000	0.001	0.001	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.140)	(0.148)	(0.148)	(0.131)
3	0.041***	0.084***	0.072***	0.051***	0.041***	0.084***	0.073***	0.052***	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.513)	(0.577)	(0.605)	(0.460)
4	0.037***	0.112***	0.111***	0.084***	0.037***	0.112***	0.112***	0.085***	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.920)	(0.807)	(0.715)	(0.919)
5	0.030***	0.129***	0.146***	0.116***	0.030***	0.130***	0.147***	0.118***	0.000	0.000	-0.001	-0.001
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.512)	(0.407)	(0.324)	(0.502)
6	0.023***	0.137***	0.175***	0.148***	0.023***	0.138***	0.176***	0.149***	0.000	0.000	-0.001	-0.001
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.677)	(0.441)	(0.328)	(0.478)

Note: *,** and *** are significant at 10, 5, and 1% levels, respectively; p-values are in parentheses.

DJCA	DJCA returns				ARMA residuals				GARCH residuals			
ε/σ	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
2	0.009***	0.019***	0.017***	0.012***	0.009***	0.018***	0.017***	0.012***	-0.001**	-0.001**	-0.001*	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.035)	(0.026)	(0.077)	(0.581)
3	0.010***	0.035***	0.040***	0.031***	0.010***	0.034***	0.040***	0.031***	-0.001**	-0.002***	-0.002**	-0.001
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.018)	(0.009)	(0.017)	(0.187)
4	0.008***	0.043***	0.061***	0.052***	0.008***	0.042***	0.060***	0.052***	0.000**	-0.002***	-0.003***	-0.002
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.022)	(0.008)	(0.010)	(0.113)
5	0.005***	0.046***	0.078***	0.073***	0.005***	0.045***	0.078***	0.072***	0.000*	-0.001**	-0.003**	-0.002
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.084)	(0.030)	(0.028)	(0.189)
6	0.003***	0.045***	0.092***	0.093***	0.003***	0.044***	0.091***	0.092***	0.000	-0.001	-0.002	-0.001
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.451)	(0.222)	(0.134)	(0.383)

Note: *,** and *** are significant at 10, 5, and 1% levels, respectively; p-values are in parentheses.

Table 4: Hurst exponents of the four Dow Jones indices

Stock returns	**DJIA**	**DJTA**	**DJUA**	**DJCA**
Original Series	0.000141	0.000846	0.000353	0.000403
Scrambled Series	0.523801	0.538644	0.521252	0.520887

ARMA residuals	**DJIA**	**DJTA**	**DJUA**	**DJCA**
Original Series	0.000112	0.000122	0.000353	0.000161
Scrambled Series	0.521610	0.504103	0.521252	0.511484

GARCH residuals	**DJIA**	**DJTA**	**DJUA**	**DJCA**
Original Series	0.000114	0.000203	0.000120	0.000170
Scrambled Series	0.534090	0.533182	0.522033	0.533036

Table 5: Correlation Dimension Analysis estimates of the four Dow Jones indices

Correlation	**Embedding Dimensions**									
Dimensions	1	2	3	4	5	6	7	8	9	10
1. DJIA returns	1.017	2.066	3.025	3.839	4.479	5.035	5.418	5.725	6.101	6.181
ARMA residuals	1.028	2.067	3.025	3.839	4.477	5.039	5.421	5.729	6.104	6.181
GARCH residuals	1.030	2.072	3.037	3.883	4.580	5.191	5.547	6.014	6.241	6.434
2. DJTA returns	1.011	2.071	3.037	3.864	4.521	4.950	5.363	5.828	5.992	6.126
ARMA residuals	1.029	2.068	3.035	3.865	4.523	4.955	5.367	5.835	5.997	6.133
GARCH residuals	1.028	2.072	3.038	3.884	4.581	5.197	5.554	6.020	6.244	6.438
3. DJUA returns	0.892	2.064	3.026	3.822	4.431	4.966	5.323	5.608	5.993	6.022
ARMA residuals	1.029	2.069	3.024	3.829	4.435	4.970	5.328	5.614	6.005	6.029
GARCH residuals	1.029	2.073	3.037	3.885	4.579	5.184	5.547	6.009	6.248	6.448
4. DJCA returns	1.017	2.071	3.035	3.805	4.515	5.060	5.495	5.769	5.986	6.306
ARMA residuals	1.029	2.066	3.039	3.806	4.519	5.064	5.501	5.776	5.994	6.316
GARCH residuals	1.027	2.071	3.033	3.867	4.567	5.181	5.625	5.991	6.345	6.388

Table 4 shows the results of the R/S analysis through the Hurst exponents. A time-series is determined by a chaotic process, the Hurst exponent would be much closer to 0.5 after scrambling the data than the one before the procedure [20]. Most of the Hurst exponents of the four Dow Jones indices returns, ARMA and GARCH residuals are way below 0.5; however, after scrambling the data, all Hurst exponents are above 0.5, which concludes that the long time-series data of the four Dow Jones indices have a persistent and a trend-reinforcing series, wherein having an upward (downward) trend in the last period, will continue to be positive (negative) in the next period. These findings support the initial findings that the four indices do not follow a random walk and an anti-persistent series. These results are again consistent with [21] when they studied the FTSE index using the Hurst test.

Table 5 presents the correlation dimension estimates for the four Dow Jones indices data series. The correlation dimension analysis is a procedure necessary for confirming chaotic behavior [26]. This research observed that as the embedding dimensions gradually increase from 1 to 10, the correlation dimension increases less quickly. This can be related to the presence of a high dimensional underlying noisy chaotic structure [27]. The tendency shows that the Dow Jones index returns, ARMA and GARCH residuals can be consistent with chaos and these findings also conforms to the study of [28] and [27] on the chaotic properties of the French CAC40 index and metal futures prices in the London Metal Exchange, respectively. These results warn technical investing analysts to be cautious in utilizing linear processes in modeling the Dow Jones indices; and provide an understanding to academicians and researchers that the long time-series of the Dow Jones indices signifies noisy chaotic tendencies and shows evidences that the EMH [15] may not be true on these set of time-series.

4 Conclusions

The study employs three tests of non-linearity and chaotic behavior, namely the BDS test, R/S analysis and correlation dimension analysis on the daily stock returns of the four Dow Jones indices - DJIA, DJUA, DJTA and DJCA. This research uses an average of 24,815 data points to correctly simulate chaos in financial time-series, following the recommendation of higher accuracy in having longer time-series data. The BDS test results rejected the presence of linearity and indicates that most of the Dow Jones indices are not *iid* or not pure random series. BDS statistics are significant for most values of ε/σ from 0.5 to 2.0, and m ranging from 2 to 6 for the filtered index returns, ARMA and GARCH residuals, except for the DJUA, wherein the filtered residuals from the GARCH model failed to reject the null hypothesis of *iid* observations. The R/S analysis demonstrates that the initial Hurst exponents of the four Dow Jones indices returns, ARMA and GARCH residuals are way below 0.5. However, after scrambling the data, all Hurst exponents are above 0.5, which concludes that the four time-series data have a persistent and a trend-reinforcing dynamics. Furthermore, the correlation dimension analysis supplements the first two initial tests, and finds that as the embedding dimensions increase from 1 to 10, the correlation dimension increases less quickly. This can be a sign of the presence of a high dimensional underlying noisy chaotic structure in the four Dow Jones indices.

These findings challenge the validity of the random walk hypothesis and the long time-series of the Dow Jones stock indices may be characterized by nonlinearity and chaotic tendencies. These results also caution the investing community in the risk of using linear processes in modeling the four Dow Jones stock indices; and give further research avenue to academicians and researchers in the chaotic tendencies of longer time-series returns. Differences in findings with regards to the non-linear characteristics can be affected by the volume of the data. The accuracy of test results improves with the increase in the length of the time series [4]. For future studies, it is suggested that researchers use larger data sets or around 5,000 or more in determining chaotic behavior of financial time-series. This minimum requirement of data may also provide some limitations on having structural breaks in the data, because some crises and possible structural breaks occur in less than 5,000 daily observations. This occurrence also limited this study in the possibility of putting structural breaks in the long time-series of the Dow Jones indices.

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