Annual Review of Chaos Theory, Bifurcations and Dynamical Systems
Vol. 18, (2019) 21-28, www.arctbds.com.

Analysis of the multicellular converter with a cubic nonlinearity

Philippe Djondiné
Department of Physics, Higher Teacher Training College of Bertoua
The University of Ngaoundéré, Cameroon
P.O. Box: 652 Bertoua
e-mail: pdjondine@yahoo.fr

Abstract: This paper reports a two multicellular converter a smooth nonlinearity, described by a cubic polynomial. Some bifurcation phenomena and chaotic attractors observed and simulated by computer for the model are presented.

Keywords: multicellular converter, cubic nonlinearity, chaos, chaotic attractor.

Manuscript accepted March 01, 2019.

1 Introduction

A multilevel converter, which includes an array of power semiconductors and capacitor voltage sources, can synthesize a desired output voltage from several levels of DC voltages as inputs. With an increasing number of DC voltage sources, the converter output voltage waveform approaches a nearly sinusoidal waveform while using a fundamental frequency switching scheme [1]. The primary advantage of multilevel converters is generating high voltage with smaller steps at the output while the power semiconductors must withstand only reduced voltages; this will results in high power quality, lower harmonic components, better electromagnetic compatibility, and lower switching losses [1, 2, 3]. One of fundamental multilevel topologies is known as Multicell Converter, including Flying Capacitor Multicell (FCM) or serial multicellular, Stacked Multicell (SM), and Cascaded Multicell (CM) converters.
The serial multicellular converter has gained substantial interest in high power systems. It allows to synthesize high voltage multilevel waveforms using low voltage power semiconductors. The first serial multicellular converter was introduced in [4]. The power

structure is an imbricated association of two or more commutation cells and flying capacitors, where the flying capacitor voltages determine the output waveform quality and the safe converter operation. However, a high number of voltage levels increases the control complexity and introduces a capacitor voltage imbalance problem [1].
In recent decades, it was discovered that most of static converters were the seat of unknown nonlinear phenomena in power electronics [5, 6, 7, 8]. It is for example the case of multicellular choppers that can exhibit unusual behaviors and sometimes chaotic behaviors. Obviously, this may generate dramatical consequences. However, the usually averaged models do not allow to predict nonlinear phenomena encountered. By nature, these models obscure the essential nonlinearities [9]. To analyses these strange behaviors, it is necessary to use a nonlinear hybrid dynamical model [7], [10]. There have been many methods for detecting chaos from order [11, 12]. Each of these methods has its advantages and drawbacks in classifying the attractors. The main purpose of the present paper is to propose a framework of chaotic behavior study for two-cell chopper with cubic nonlinearity load. The paper is structured as follows. Section 2 deals with the modeling process. The electronic structure of the serial multicell chopper is addressed and the appropriate mathematical model is derived to describe the dynamics of the chopper. Two cells chopper modeling is then considered. Chaotic behavior and simulation results are presented in Section 3. Finally, some conclusion and remarks are reported in section 4.

2 Studied multicellular: A general model of description

The general structure of the studied multicellular converter is presented in figure 1. It is composed of p-cells. Each cell contains two complementary power electronic components controlled by a binary switch. That means that if the upper switch of the k^{th} cell is closed $u_k = 1$ and the lower switch is open. The multicellular converter cells are associated in

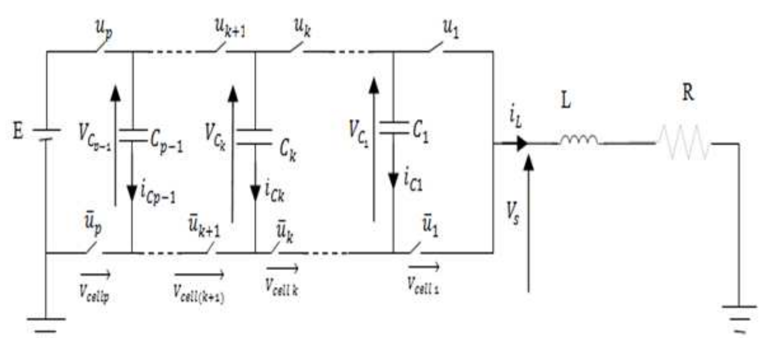

Figure 1: Studied p-cells converter.

series with a RL load and the cells are separated by capacitors that can be considered as continuous voltage sources [13, 14]. Thus, the converter has $p-1$ floating voltage sources. In order to ensure normal operations, it is necessary to guarantee a balanced distribution of the floating voltages $V_{C_k} = k\frac{E}{p}$. The output voltage V_S can attend p voltage levels

$(\frac{E}{p}, ..., (p-1)\frac{E}{p}, E)$[14].
Note that the chopper, which has a purely dissipative load, cannot generate a chaotic behavior. Nevertheless, it is well known from [15] that power converter, when it is connected to nonlinear load may have a chaotic behavior. In this paper, we study the overlapping operation of a converter with two-cells (figure 2). Its function is to supply a passive load (RL) in series with another nonlinear load connected in parallel with a capacitor (figure 2) [9] The state equations describing the circuit are as follows:

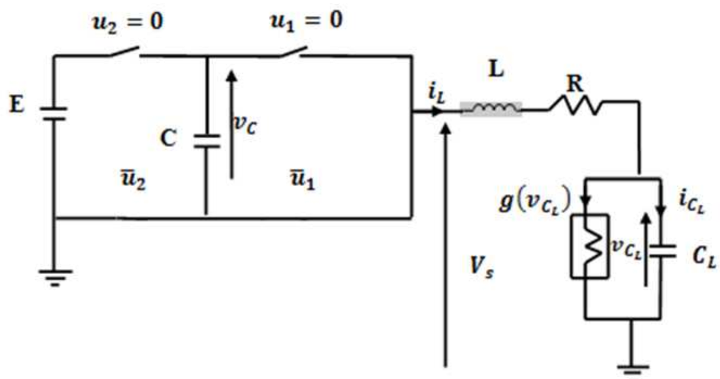

Figure 2: Two-cells converter.

$$\begin{cases} L\frac{di_L}{dt} & = (u_2 - u_1)v_C - v_{C_l} - Ri_L + u_2E \\ C\frac{dv_C}{dt} & = (u_2 - u_1)i_L \\ C_l\frac{dv_{C_l}}{dt} & = i_L - g(v_{C_l}) \end{cases} \tag{1}$$

where $g(v_{C_l})$ is a piecewise-linear function defined by

$$g(v_{C_l}) = G_b v_{C_l} + \frac{1}{2}(G_a - G_b)(|v_{C_l} + 1| - |v_{C_l}|)$$

and v_C, v_{C_l}, and i_L, denote voltage across C, voltage across C_l, and current through L, respectively. Figure 3 shows the chaotic attractor observed by solving (1) with $C = 0.1\mu F, C_l = 40\mu F, L = 50mH, R = 10\Omega, E = 100V, G_a = -1.5, G_b = 0.5$ Figure 3(a) - (c) are the projections of the attractor onto the (i_L, v_C, v)-plane, (v, v_C)-plane, and the nonlinear function, respectively.
Let's replace the piecewise -linear function with a cubic polynomial function and observe the behavior of the converter (figure 4):

$$g(v_{C_l}) = a.v_{C_l} + b.{v_{C_l}}^3$$

with $a = -0.599, b = 0.0218$

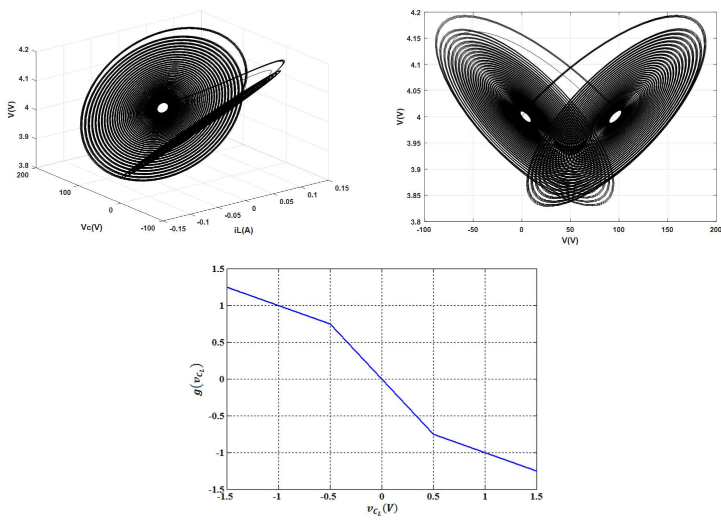

Figure 3: Attractor of of equation 1.

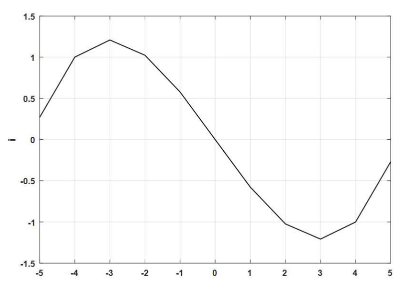

Figure 4: Characteristic $i - v$ of nonlinear load with a cubic nonlinearity

3 Bifurcation and chaos in the two cell converter with a cubic nonlinearity

The state equations for two cells converter in Figure 1 with a cubic nonlinearity are as follows

$$\begin{cases} L\frac{di_L}{dt} &= (u_2 - u_1)v_C - v_{C_l} - Ri_L + u_2E \\ C\frac{dv_C}{dt} &= (u_2 - u_1)i_L \\ C_l\frac{dv_{C_l}}{dt} &= i_L - g(v_{C_l}) \end{cases} \tag{2}$$

where

$$g(v_{C_l}) = a.v_{C_l} + b.{v_{C_l}}^3$$

with $a = -0.599, b = 0.0218$
Rescaling equation (2) as $i_L = x_1G, v_C = x_2, v_{C_l} = x_3, G = \frac{1}{R}, t = \frac{C}{G}\tau$ and then redefining τ as t the following set of normalised equation are obtained.

$$\begin{cases} \dot{x}_1 &= \beta(-\gamma x_1 + \varepsilon x_2 - x_3) + \alpha E \\ \dot{x}_2 &= \varepsilon x_1 \\ \dot{x}_3 &= p(x_1 - g(x_3)) \end{cases} \tag{3}$$

where $\varepsilon = u_2 - u_1, p = \frac{C}{C_l}, \beta = \frac{C}{LG^2}, \gamma = RG, \alpha = \beta u_2$. The circuit parameters used are rescaled as : $p = 25.10^{-4}, \beta = 2.10^{-4}, \gamma = 1$.
The system has the following equilibrium:
Case 1 : $\varepsilon = 1$, i.e., $u_2 = 1, \quad u_1 = 0, \quad \Rightarrow \alpha = \beta$

$$E_1 = \begin{pmatrix} 0 \\ -E \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1500 \\ 0 \end{pmatrix}; E_2 = \begin{pmatrix} 0 \\ \sqrt{-\frac{a}{b}} - E \\ \sqrt{-\frac{a}{b}} \end{pmatrix} = \begin{pmatrix} 0 \\ -1497.7581 \\ 5.2418 \end{pmatrix},$$

$$E_3 = \begin{pmatrix} 0 \\ \sqrt{-\frac{a}{b}} - E \\ -\sqrt{-\frac{a}{b}} \end{pmatrix} = \begin{pmatrix} 0 \\ -1505.2418 \\ -5.2418 \end{pmatrix}.$$

Case 2 : $\varepsilon = -1$, i.e., $u_2 = 0, \quad u_0 = 0, \quad \Rightarrow \alpha = 0$

$$E_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; E_5 = \begin{pmatrix} 0 \\ -\sqrt{-\frac{a}{b}} \\ \sqrt{-\frac{a}{b}} \end{pmatrix} = \begin{pmatrix} 0 \\ -5.2418 \\ 5.2418 \end{pmatrix}; E_6 = \begin{pmatrix} 0 \\ \sqrt{-\frac{a}{b}} \\ -\sqrt{-\frac{a}{b}} \end{pmatrix} = \begin{pmatrix} 0 \\ 5.2418 \\ -5.2418 \end{pmatrix}$$

Case 3 : $\varepsilon = 0, i.e \quad u_2 = 1, \quad u_0 = 1, \quad \Rightarrow \alpha = \beta$

$$E_7 = \begin{pmatrix} 1459.2 \\ x_2 \\ 40.8278 \end{pmatrix}; E_8 = \begin{pmatrix} 1520.4 + 35.613i \\ x_2 \\ -20.4139 - 35.6170i \end{pmatrix}; E_9 = \begin{pmatrix} 1520.4 - 35.613i \\ x_2 \\ -20.4139 + 35.617i \end{pmatrix}$$

Let us study the stability of different equilibrium points. The Jacobian matrix is defined as

$$J = \begin{bmatrix} -\beta\gamma & \beta\varepsilon & -\beta \\ \varepsilon & 0 & 0 \\ p & 0 & -ap - 3apx_3^2 \end{bmatrix} = \begin{bmatrix} -2.10^{-4} & -2.10^{-4}\varepsilon & -2.10^{-4} \\ \varepsilon & 0 & 0 \\ 25.10^{-4} & 0 & -14.975.10^{-4} - 44.925.10^{-4}x_3^2 \end{bmatrix}$$

- **For** $\varepsilon = \pm 1$
  The eigenvalues are:

  i **If** $x_3 = 0$; $\lambda_1 = -0.0001 + 0.0142i; \lambda_2 = -0.0001 - 0.0142i; \lambda_3 = -0.0015$. All the eigenvalues have their real negative part. Therefore, the equilibrium $E_1, E_2, E_3, E_4, E_5 \quad and \quad E_6$ are asymptotically stable.

  ii **If** $x_3 = \pm\sqrt{-\frac{a}{b}} = \pm 5.2418$; $\lambda_1 = -0.0001 + 0.0141i; \lambda_2 = -0.0001 - 0.0141i; \lambda_3 = -0.001249$. All the eigenvalues have their real negative part. Therefore, the equilibrium $E_1, E_2, E_3, E_4, E_5 \quad and \quad E_6$ are asymptotically stable.

- **For** $\varepsilon = 0$
  The eigenvalues are:

  i **If** $x_3 = 0$; $\lambda_1 = -0.8488.10^{-3} + 0.2813.10^{-3}i; \lambda_2 = -0.8488 - 0.2813.10^{-3}i; \lambda_3 = 0$. Here λ_1 and λ_2 have their real negatives part and λ_3 is null. Therefore, the equilibrium $E_7, E_8, \quad and \quad E_9$ are unstable.

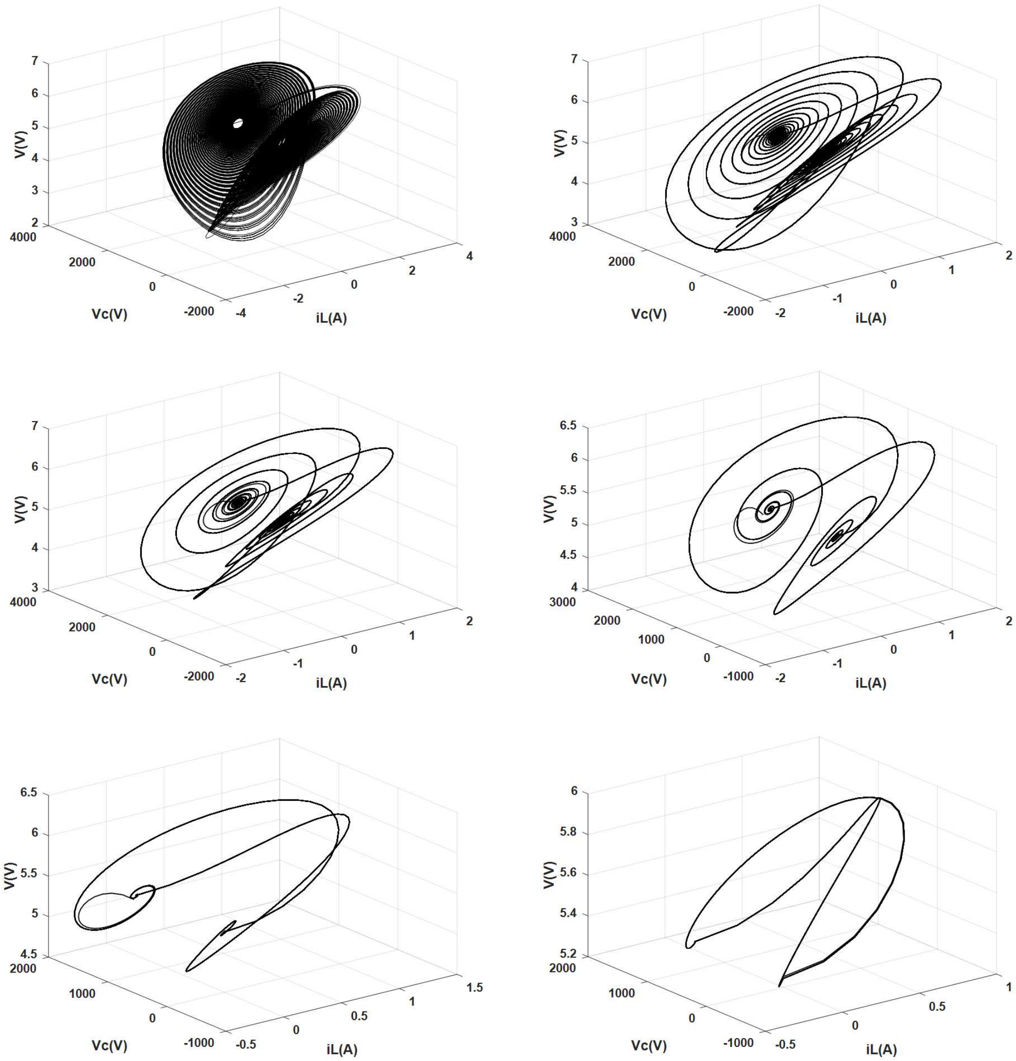

Figure 5: Attractor of equation 1 with a cubic nonlinearity

ii **If** $x_3 = \pm\sqrt{-\frac{a}{b}} = \pm 5.2418;\ \lambda_1 = -0.0002; \lambda_2 = -0.1249; \lambda_3 = 0$. Here λ_1 and λ_2 are the negative reals and λ_3 is null. Therefore, the equilibrium $E_7, E_8, \quad and \quad E_9$ are unstable.

Figure 5(a)-(f) shows the bifurcation sequence with respect to R and the chaotic attractors Note from these pictures that there is a period-doubling route to chaos similar to that observed from two cell converter with a piecewise-linear function [16].

4 Conclusion

It is well known that the multicellular converter connected to a nonlinear load can exhibit a wide variety of nonlinear behaviors. Though most of the interesting chaotic phenomena can be described by multicellular converter with a piecewise-linear function. The smooth nonlinearity with a cubic polynomial presented in this paper contributes a robust model.

References

[1] J. Rodriguez, J.S. Lai and F.Z. Peng, Multilevel inverters: A survey of topologies, controls, and applications. IEEE Trans. Ind. Electron 2002; 49: 724–738.

[2] A. Nabae, I. Takahashi and H. Akagi, A New Neutral-Point-Clamped PWM Inverter, IEEE Trans. Ind. Appl. 1981; 17: 518–523.

[3] M. Sadeghi, S.H. Hosseini, R. Alizadeh and M.R. Banaei, New Mixed Stacked Multicell Converter with Interesting Advantages, IET power electronic (PEL) 2012; 5: 1298–1304.

[4] T. A. Meynard and H. Foch, Multilevel choppers for high voltage applications, EPE Journal 1992; 2: 45–50.

[5] F. Defay, A.M. Llor and M. Fadel, A predictive control with flying capacitor balancing of a multicell active power filter, IEEE Trans. Ind. Electron 2008; 55: 3212–3220.

[6] M. Di Bernardo and K. T. Chi, Bifurcation and chaos in power electronics: an overview in nonlinear dynamics in engineering, Edited by Prof. G. Chen and T. Ueta, World Scientific, 2002; 317–340.

[7] M. Ghanes, M. Trabelsi, X.L. Shi, J.P. Barbot, J.M. Retif and K. Busawon High gain observer for a three-cell chopper: Design and experimental results, Internat. J. Robust Nonlinear Control 2013; 24: 1090–1103.

[8] J. I. Leon, R. Portillo, S. Vazquez, J.J. Padilla, L.G. Franquelo and J. M. Carrasco Simple unified approach to develop a time-domain modulation strategy for single-phase multilevel choppers, IEEE Trans. Ind. Electron. 2008; 55: 3239–3248.

[9] P. Djondiné, R. He, M. Ghanes and J-P. Barbot, Chaotic behavior study for serial multicellular chopper connected to nonlinear load , Proceedings of the 3rd International Conference on Systems and Control, Algiers, Algeria, October 29-31, 2013.

[10] J.P. Barbot, H. Saadaoui and N. Manamanni, Nonlinear observer for autonomous switching systems with jump, Nonlinear Anal. Hybrid Syst. 2007; 1: 537–547.

[11] G. Chen and X. Dong, From chaos to order: Methodologies, perspectives and applications, World Scientific Series on Nonlinear Science Series A, 1998; 24, 776 pages.

[12] G. Contopoulos, Order and chaos in dynamical astronomy, Springer Verlag, 2002.

[13] Sadigh, A.K., Hosseini, S.H., and G.B. Gharehpetian, Double flying capacitor multicell converter based on modified phase-shifted pulse width modulation, IEEE Transactions on Power Electronics, 2010; 259(6): 1517–1526.

[14] K. Benmansour, A. Benalia, M. Djemai, J. De Leon. Hybrid control of a multicellular converter. Nonlinear Analysis: Hybrid Systems, 2007; 1(1): 16-19.

[15] T. A. Meynard, M. Fadel, and N. Aouda, Modeling of Multilevel Converters, IEEE Trans. Ind. Electronics, 1997; 3(44): 356–64.

[16] Philippe Djondiné, Jean-Pierre Barbot, Nonlinear dynamic of the multicellular chopper, Int. J. Nonlinear Anal. Appl. 2018; 9(2): 21–31.