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True and False Chaotic Attractors in a 3-D Lorenz-type System

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Abstract: In some known literatures those authors have analyzed the Yang system, $\dot{x} = a(y-x)$, $\dot{y} = cx - xz$, $\dot{z} = -bz + xy$, containing three independent parameters. They think that they have found the system to have two interesting chaotic attractors (called as Yang–Chen attractor) when $(a, b, c) = (10, \frac{8}{3}, 16)$ and $(a, b, c) = (35, 3, 35)$, respectively. However, by further analysis and Matlab simulation, we show that the two Yang–Chen chaotic attractors found are actually pseudo ones. In fact, the two attractors are locally asymptotically stable equilibria. Further, we present the values of parameters for this system to really generate chaotic attractor. Accordingly, we find a new attractor in the Yang system co-existing with one saddle and two stable node-foci.

Keywords: Yang system, Yang-Chen attractor, pseudo chaotic attractor, Lyapunov exponent.

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1 Introduction

In the paper [1] the authors considered a Lorenz-type system with three parameters given by

$$\begin{cases} \dot{x} &= a(y-x), \\ \dot{y} &= cx - xz, \\ \dot{z} &= -bz + xy, \end{cases} \tag{1}$$

where $a > 0$, $b > 0$ and $c \in \mathbb{R}$, named as Yang system later. Its rich dynamics was studied in [1] from the points of views of system equilibria, Lyapunov exponents, bifurcations, fractional dimension and Poincaré maps. Afterwards, Yang system was further explored in [2] in sight of bifurcation of equilibria, non-isolated equilibria and the existence of homoclinic and heteroclinic orbits when $b \in \mathbb{R}$. All these results are also illustrated from numerical simulations.

The reason of why they consider this system is, they think that, this system is very interesting because it "is a chaotic system with one saddle and two stable node-foci in a simple three-dimensional (3D) autonomous system." ([1], Abstract, p. 1393). Especially, they think, when $(a, b, c) = (10, \frac{8}{3}, 16)$ and $(a, b, c) = (35, 3, 35)$, respectively, they found that this system has two interesting chaotic attractors, called as Yang-Chen attractor later.

It is well known that a remarkable nature for the occurrence of chaos in a given system is that this system has at least one positive Lyapunov exponent for given parameters. Unfortunately, for the two groups of parameter values, as what we derive, this system has no such properties. So, the two chaotic attractors found are in deed pseudo chaotic attractors. These can be shown as follows.

For the known given parameters $(a, b, c) = (10, \frac{8}{3}, 16)$, taking the initial values $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$, we obtain its Lyapunov exponents $L_1 = -0.049684$, $L_2 = -0.054422$ and $L_3 = -12.561778$, shown in Figure 1, all of which are negative! This implies that this system can not generate chaos under this condition.

For the phase portrait of the Yang system with such parameter values in $\mathbb{R}^3$, or the first pseudo Yang-Chen attractor, see Figure 1. The graphs for the corresponding time series of variables x, y, z and the projections in phase plane $x-y, z-x, z-y$, refer to Figure 3–Figure 4 respectively. Figure 3 clearly displays that the variables x, y and z eventually are stable. So, the system is non–chaotic for $(a, b, c) = (10, \frac{8}{3}, 16)$.

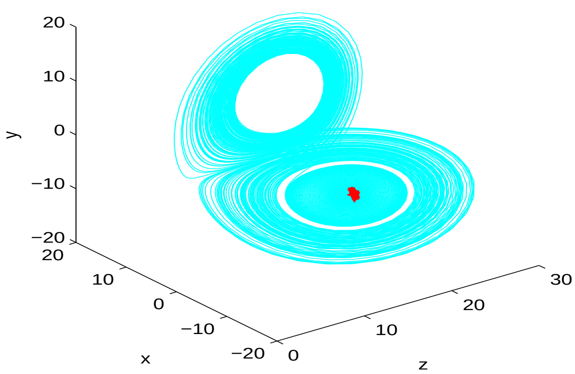


Figure 1: The first pseudo attractor in R^3 with $(a, b, c) = (10, \frac{8}{3}, 16)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$, time of integration: $[0, 1e4]$. The figure suggests the solutions of system (1) tend to its equilibrium point eventually under the first given parameter and initial condition.

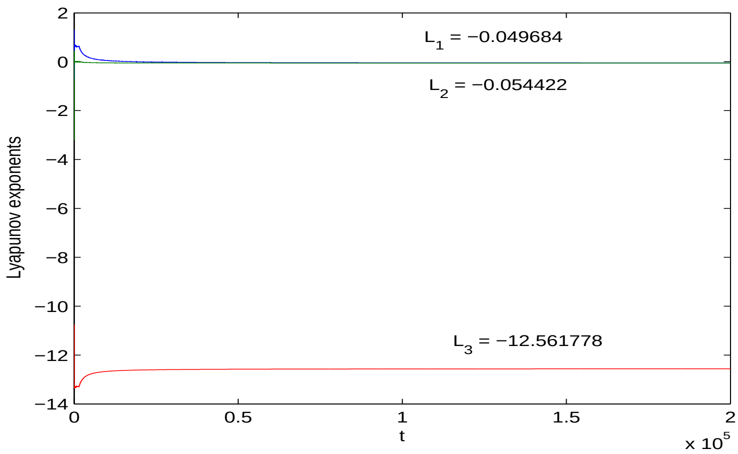
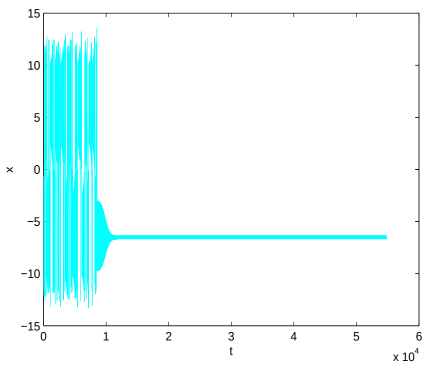
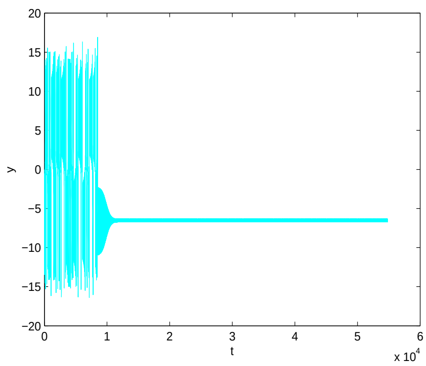


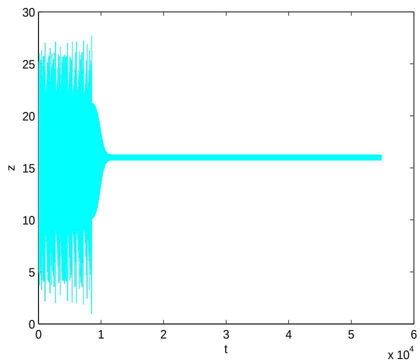
Figure 2: Lyapunov Exponents for the first pseudo attractor when $(a, b, c) = (10, \frac{8}{3}, 16)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$, time of integration: $[0, 2 \times 1e5]$. All of them are less than zero. This figure indicates system (1) is not chaotic under the given parameter and initial condition.



(a) Time series for x

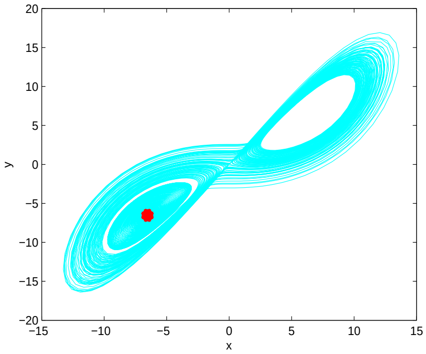


(b) Time series for y

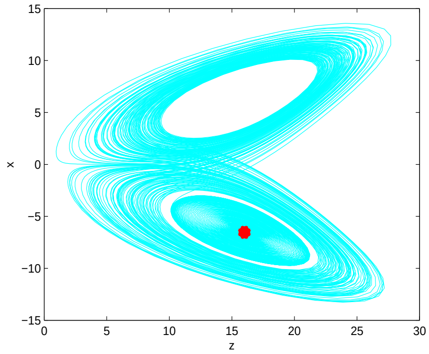


(c) Time series for z

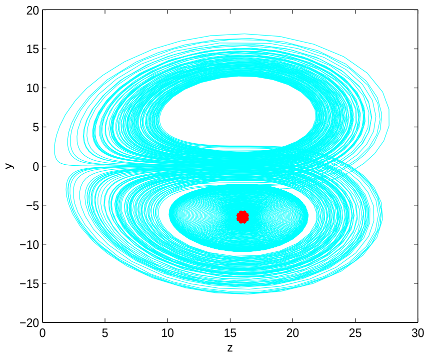
Figure 3: Time series of the first pseudo attractor when $(a, b, c) = (10, \frac{8}{3}, 16)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$. This figure clearly illustrates that the variables x, y and z tend to its equilibrium point for the given parameter and initial condition.



(a) Projection in plane xy



(b) Projection in plane zx



(c) Projection in plane zy

Figure 4: Projections of the first pseudo attractor when $(a, b, c) = (10, \frac{8}{3}, 16)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$.

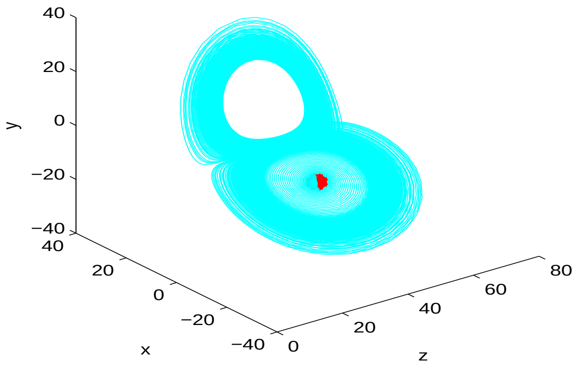


Figure 5: The second pseudo attractor in R^3 with $(a, b, c) = (35, 3, 35)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$, time of integration: $[0, 1e4]$. The figure suggests the solutions of system (1) tend to its equilibrium point eventually under the second given parameter and initial condition.

For the given parameters $(a, b, c) = (35, 3, 35)$, still taking $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$, three negative Lyapunov exponents are also obtained (see Table 1). So, this system can

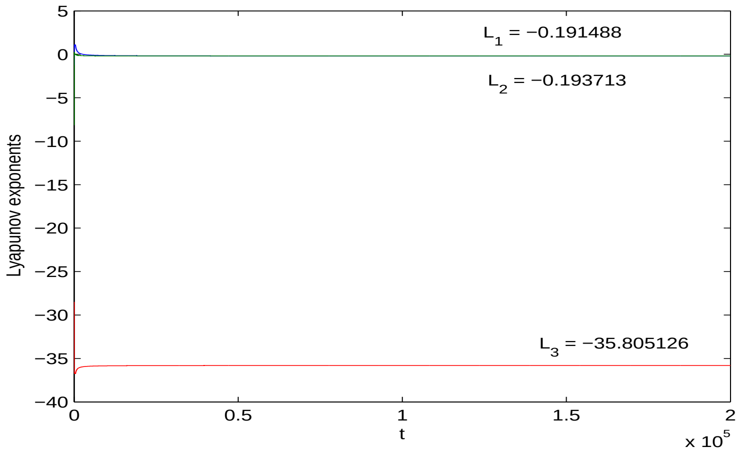
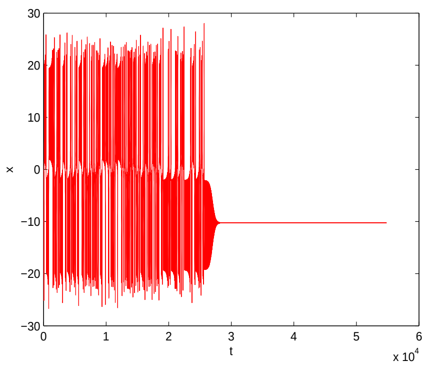
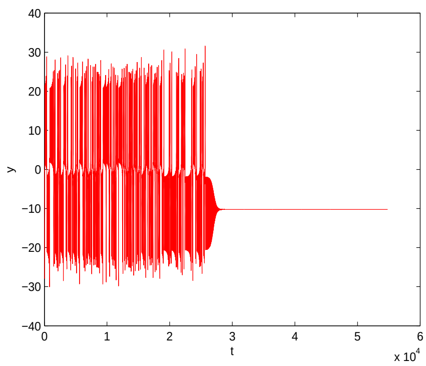


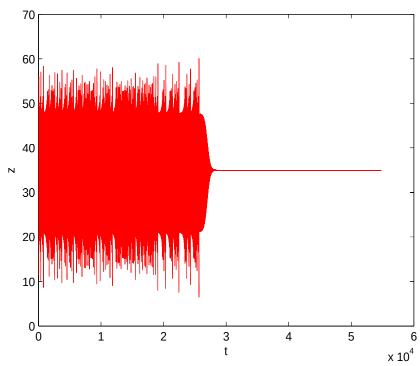
Figure 6: Lyapunov Exponents for the second pseudo attractor when $(a, b, c) = (35, 3, 35)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$, time of integration: $[0, 2 \times 1e5]$. All of them are less than zero. This figure indicates system (1) is not chaotic under the second given parameter and initial condition.



(a) Time series for x

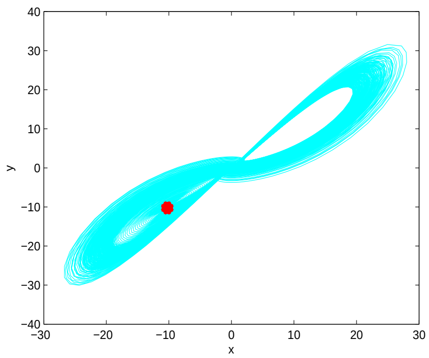


(b) Time series for y

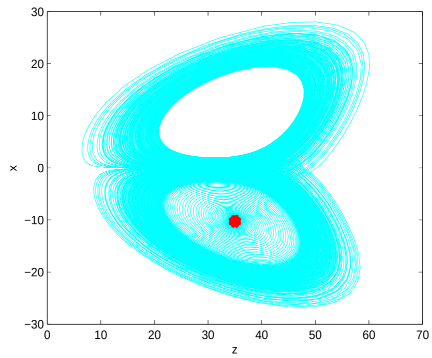


(c) Time series for z

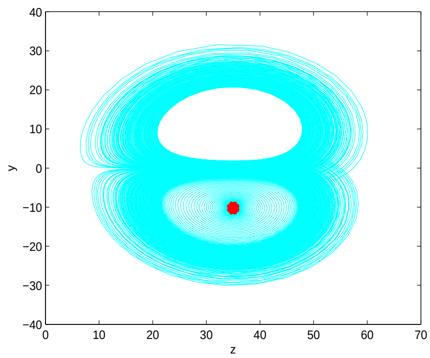
Figure 7: Time series of the second pseudo attractor when $(a, b, c) = (35, 3, 35)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$. This figure clearly shows that the variables x, y and z tends to its equilibrium point for the second given parameter and initial condition.



(a) Projection in plane xy



(b) Projection in plane zx



(c) Projection in plane zy

Figure 8: Projections of the second Yang attractor when $(a, b, c) = (35, 3, 35)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$.

not generate chaos under this condition, either. For the graph illustrations for our results, see the following Figure 5–Figure 8.

It is known to all that for a given system with several parameters, its dynamical properties are closely related to the values of its parameters taking. Naturally, one may ask "Whether does Yang system have this kind of novel and interesting chaotic attractor-under the condition of the system with one saddle and two stable foci–node?". The answer is yes! In fact, the author in [3] stated that the following Lorenz–like system

$$\begin{cases} \dot{x} &= a(y - x), \\ \dot{y} &= a(bx - xz), \\ \dot{z} &= -cz + xy, \end{cases} \tag{2}$$

where $(a, b, c) \in \mathbb{R}^3$, has such chaotic attractors when $(a, b, c) = (1, 1, 0.08)$ and $(x_0, y_0, z_0) = (0, 0.5, 1.5)$ or $(-0.3, -0.3, 2)$ ([3], Figure 6, caption, p. 3499). Further, its Lyapunov exponents are $L_1 = 0.030175$, $L_2 = -0.000004$ and $L_3 = -1.110171$ (see Figure 9) for $(x_0, y_0, z_0) = (0, 0.5, 1.5)$, and $L_1 = 0.030085$, $L_2 = -0.000009$ and $L_3 = -1.110076$ (see Figure 10) for $(x_0, y_0, z_0) = (-0.3, -0.3, 2)$; so system (2) are chaotic attractors when taking both of the two initial conditions. Thus, correspondingly, Yang system has this kind of chaotic attractor when $(a, b, c) = (1, 0.08, 1)$ and the same two initial conditions as above.

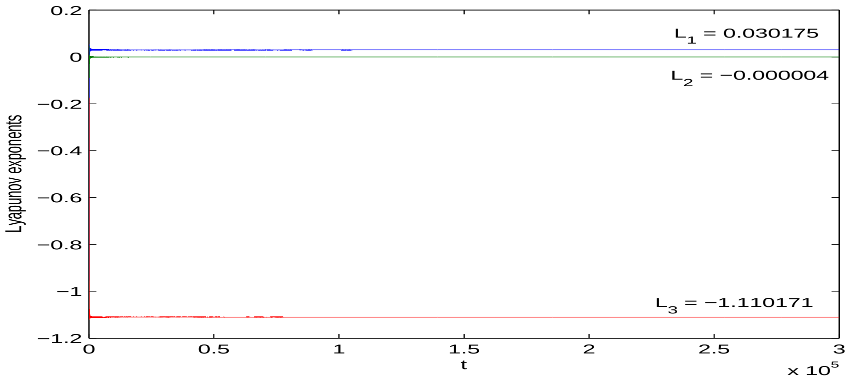


Figure 9: Lyapunov Exponents for system (2) when $(a, b, c) = (1, 1, 0.08)$ and $(x_0, y_0, z_0) = (0, 0.5, 1.5)$, time of integration: $[0, 3 \times 1e5]$. This figure indicates system (2) is chaotic under the first given initial condition.

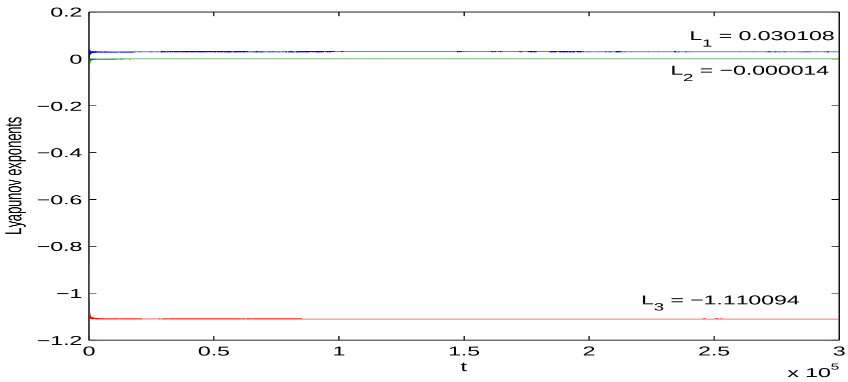


Figure 10: Lyapunov Exponents for system (2) when $(a, b, c) = (1, 1, 0.08)$ and $(x_0, y_0, z_0) = (-0.3, -0.3, 2)$, time of integration: $[0, 3 \times 1e5]$. This figure indicates system (2) is chaotic under the third given initial condition, too.

In addition, we find that, Yang system has also this kind of chaotic attractor when $(a, b, c) = (6, 3, 17)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$. For the numerical simulation results, refer to the following Figure 11-Figure 14.

For the sake of convenience of comparison, the equilibria, eigenvalues and Lyapunov exponents for Yang system with different parameter values (a, b, c) are list in the following Table 1.

To show that the two Yang-Chen chaotic attractors found are pseudo ones and find the new and true attractor coexistence with a saddle and two stable node-foci is just the aim of this letter.

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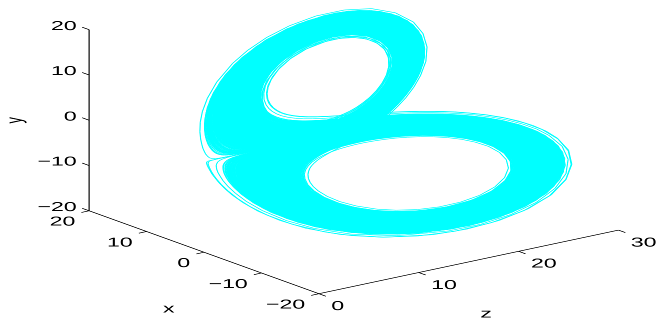


Figure 11: Chaotic attractor in $\mathbb{R}^3$ with $(a, b, c) = (6, 3, 17)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$ for Yang system, time of integration: $[0, 1e4]$.

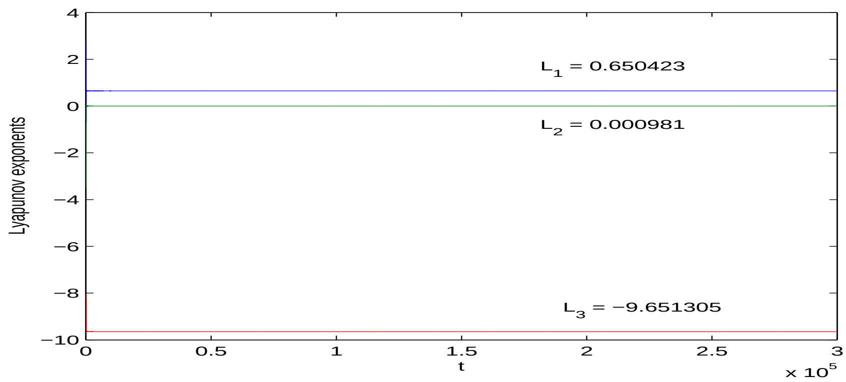


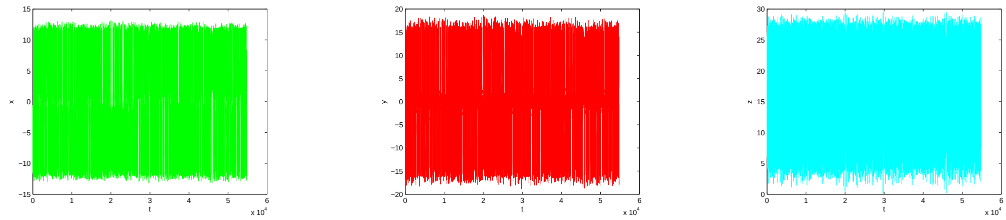
Figure 12: Lyapunov Exponents for Yang system when $(a, b, c) = (6, 3, 17)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$, time of integration: $[0, 3 \times 1e5]$. This figure indicates system (2) is chaotic under the first given parameter and initial condition.

References

[1] Q. Yang and G. Chen *A chaotic system with one saddle and two stable node-foci*, Internat. J. Bifur. Chaos Appl. Sci. Engrg., 18 pp. 1393-1414, (2008.)

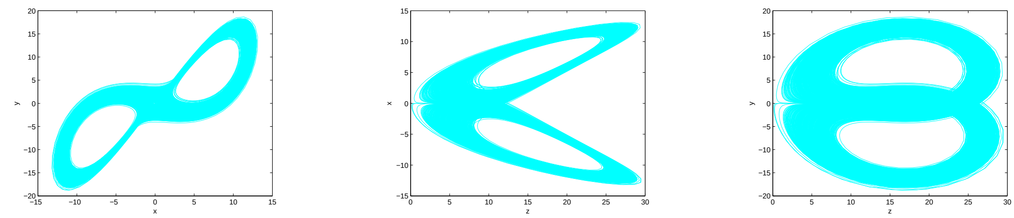
[2] Y. Liu and Q. Yang *Dynamics of a new Lorenz-like chaotic system*, Nonlinear Anal. Real World Appl., 11 pp. 2653-2572, (2010.)

[3] F. S. Dias, L. F. Mello and J. Zhang *Nonlinear analysis in a Lorenz-like system*, Nonlinear Anal. Real World Appl., 11 pp. 3491-3500, (2010.)



(a) Time series for x (b) Time series for y (c) Time series for z

Figure 13: Time series of Yang system when $(a, b, c) = (6, 3, 17)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$. This figure clearly illustrates that all variables x, y and z have broad spectrum for the given parameter and initial condition.



(a) Projection in plane $x - y$ (b) Projection in plane $z - x$ (c) Projection in plane $z - y$

Figure 14: Projections of Yang system when $(a, b, c) = (6, 3, 17)$ and $(x_0, y_0, z_0) = (1.15, 3.5, 3.3)$.

Table 1: Equilibria, eigenvalues and Lyapunov exponents for Yang system.

(a, b, c)	Equilibria	Eigenvalues	Lyapunov Exponents
$(10, \frac{8}{3}, 16)$	$(\pm 6.5320, \pm 6.5320, 16)$	$\lambda_1 = -12.5570$ $\lambda_2 = -0.0548 + 8.2434i$ $\lambda_3 = -0.0548 - 8.2434i$	$L_1 = -0.049684$ $L_2 = -0.054422$ $L_3 = -12.561778$
$(35, 3, 35)$	$(\pm 10.2470, \pm 10.2470, 35)$	$\lambda_1 = -37.6122$ $\lambda_2 = -0.1939 + 13.9778i$ $\lambda_3 = -0.1939 - 13.9778i$	$L_1 = -0.191488$ $L_2 = -0.193713$ $L_3 = -35.805126$
$(6, 3, 17)$	$(\pm 7.1414, \pm 7.1414, 17)$	$\lambda_1 = -8.9396$ $\lambda_2 = -0.0302 + 8.2740i$ $\lambda_3 = -0.0302 - 8.2740i$	$L_1 = 0.650423$ $L_2 = 0.000981$ $L_3 = -9.651305$